Gauge Models in Modified Triplectic Quantization

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We apply the modified triplectic formalism for quantization of several popular gauge models. Thus we consider the model of non-abelian antisymmetric tensor field, the model of W_2-gravity, and the model of two-dimensional gravity with dynamical torsion. For the models in question we obtain explicit solutions of the generating equations that determine the quantum action and the gauge-fixing functional. Using these solutions, we construct the vacuum functional and obtain the correponding transformations of extended BRST symmetry.

1. Introduction

In recent years the development of covariant quantization rules for general gauge theories on the basis of extended BRST symmetry has become increasingly popular[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11].

The realization of the principle of extended BRST symmetry, combining the BRST [12] and anti-BRST [13] transformations, naturally implies the tendency of unified treatment of auxiliary variables that serve to parametrize the gauge in the functional integral and those that enter the quantum action determined by the corresponding generating equations. Basically, the above tendency manifests itself in enlarging the configuration space of the quantum action with auxiliary gauge-fixing variables (see, e.g., [1, 2, 3]). Recently, however, it has been strengthened by extending the concept of generating equations to the case of introducing the gauge [2, 6].

The method of $Sp(2)$ covariant quantization [1] was one of the first to provide a realization of extended BRST symmetry for general gauge theories, i.e. theories of any stage of reducibility with a closed or open algebra of gauge transformations. The complete configuration space ϕ^A of a gauge theory, considered in this approach, is constructed by the rules of the BV quantization [14] and consists of the initial classical fields supplemented by the pyramids of auxiliary variables, i.e. ghosts, antighosts and Lagrange multipliers, according to the corresponding stage of reducibility. Even though the above auxiliary variables originally [14] play different roles in the construction of the quantum theory, their consideration within the $Sp(2)$ covariant formalism allows to achieve a remarkable uniformity of description. Namely, in the framework of the $Sp(2)$ covariant approach, the pyramids of ghosts are combined with the corresponding pyramids of antighosts and Lagrange multipliers into irreducible representations of the $Sp(2)$ group, which form completely symmetric $Sp(2)$ tensors and enter the quantum theory on equal footing in terms of both the quantum action and the introduction of gauge. The quantum action of the $Sp(2)$

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covariant formalism depends on an extended set of variables, including, besides the fields ϕ^A, also the sets of antifields ϕ^*_{Aa} and $\bar{\phi}_A$. Note that in the case of linear dependence of the quantum action on ϕ^*_{Aa} and $\bar{\phi}_A$ they may be interpreted as sources of extended BRST transformations and sources of mixed BRST and anti-BRST transformations, respectively.

In [3], a closed superfield form of the $Sp(2)$ covariant rules was proposed. This approach allows to combine all the variables of the $Sp(2)$ covariant formalism, namely, the fields and antifieds $(\phi^A, \phi^*_{Aa}, \bar{\phi}_A)$ that enter the quantum action, the auxiliary variables (π^{Aa}, λ^A) that serve to parametrize the gauge, and, finally, the sources J_A to the fields ϕ^A, into superfields Φ^A and supersources $\bar{\Phi}_A$ defined in a superspace with two scalar Grassmann coordinates. The quantum action of that theory is defined as a functional of superfields and supersources, which makes it possible to realize the transformations of extended BRST symmetry in terms of supertranslations along the Grassmann coordinates.

Moreover, in the recent paper [4] a superspace approach to massive gauge theories was developed, where it was shown that mass-dependent BRST and anti-BRST operators generate special conformal transformations coupled to translations in a 2-dimensional superspace, which implies that the $Sp(2)$ symmetry (including ghost number conservation) and the symmetry underlying the new ghost number conservation can be realized as symplectic rotations and dilatations in superspace.

In the framework of the triplectic formalism [2] a modification of the $Sp(2)$ covariant approach was proposed, based on a different extension of the configuration space of the quantum action. Namely, it was suggested to consider the auxiliary fields π^{Aa} as variables anticanonically conjugated to the antifields $\bar{\phi}_A$ with the corresponding redefinition of the extended antibrackets [1] which appear in the generating equations for the quantum action. Another feature of the triplectic formalism is that the gauge-fixed part of the action in the functional integral is determined by generating equations formally similar to the equations that describe the quantum action. The entire set of variables necessary for the construction of the vacuum functional in the triplectic formalism coincides with the corresponding set of the $Sp(2)$ covariant approach and is composed by the fields (ϕ^A, ϕ^*_{Aa}) and $(\pi^{Aa}, \bar{\phi}_A)$ anticanonically conjugated in the sense of modified antibrackets, as well as by the remaining auxiliary fields λ^A that serve to parametrize the gauge-fixing functional.

In the recent paper [6] a modification of the triplectic formalism was proposed. Thus, while retaining the space of variables of the triplectic formalism and accepting the idea of imposing generating equations on both the quantum action and the gauge-fixing functional, it was suggested in [6] to modify the system of these equations, as well as to adjust the definition of the vacuum functional, in order to ensure the correct boundary conditions for the quantum action. This allows to take into account the information contained in the classical action, namely, to guarantee that the quantum action reduces to the classical action at vanishing antifields and quantum corrections, i.e.,

$$S|_{\Phi^*=\bar{\phi}=\hbar=0} = S_0,$$

which implies that the classical action of a theory satisfies the generating equations for the quantum action,

$$\frac{1}{2}(S,S)^a + V^a S = i\hbar\Delta^a S, \tag{1.1}$$

in complete analogy with earlier quantization schemes, and in contrast to the original triplectic formalism [2]. The gauge fixing functional of the modified triplectic formalism satisfies similar generating equations,

$$\frac{1}{2}(X,X)^a - U^a X = i\hbar\Delta^a X. \tag{1.2}$$

The above systems of generating equations are expressed in terms of the differential operators

$$\begin{aligned}
\Delta^a &= (-1)^{\varepsilon_A}\frac{\delta_l}{\delta\phi^A}\frac{\delta}{\delta\phi^*_{Aa}} + (-1)^{\varepsilon_A+1}\varepsilon^{ab}\frac{\delta_l}{\delta\pi^{Ab}}\frac{\delta}{\delta\bar{\phi}_A}\\
V^a &= \varepsilon^{ab}\phi^*_A\frac{\delta}{\delta\bar{\phi}_{Ab}}\\
U^a &= (-1)^{\varepsilon_A+1}\pi^{Aa}\frac{\delta_l}{\delta\phi^A},
\end{aligned}$$

where the derivatives with respect to the antifields are taken from the left, and ε^{ab} is the antisymmetric tensor with the normalization $\varepsilon^{12}=1$. The operators V^a and U^a are closely related to operators which were introduced earlier in the framework of the superfield formalism [3], and which have a clear geometrical meaning as generators of supertranslations in superspace. The extended antibrackets $(\ ,\ )^a$ used in Eqs. (1.1), (1.2) are generated in usual way [1, 2, 5] by acting of the operators Δ^a on product of two arbitrary functionals.

Given the quantum action S and the gauge-fixing functional X, the vacuum functional Z in the framework of the modifed triplectic quantization [6] is defined by the rule

$$Z = \int d\phi\, d\phi^* d\pi\, d\bar{\phi}\, d\lambda \exp\left\{\frac{i}{\hbar}\left(S + X + \phi^*_{Aa}\pi^{Aa}\right)\right\}. \tag{1.3}$$

In this paper we apply the prescriptions of the modified triplectic formalism for quantization of several gauge theory models.

In Section 2, we consider the model of antisymmetric tensor field suggested by Freedman and Townsend [15]. The Freedman–Townsend model is an abelian gauge theory of first stage reducibility. The corresponding complete configuration space is constructed by the rules of the $Sp(2)$ covariant formalism [1] for reducible gauge theories. In the case of the Freedman–Townsend model, the generating equations (1.1) that determine the quantum action in the framework of the modified triplectic formalism can be solved exactly, which allows one to obtain the exact form of the vacuum functional and the transformations of extended BRST symmetry.

In Sections 3 and 4 we consider the gauge models of W_2-gravity [16] and of two-dimensional gravity with dynamical torsion [17], respectively. Both these models are examples of irreducible gauge theories with a closed algebra, and their configuration spaces are constructed by the rules of the $Sp(2)$ covariant quantization for irreducible theories. To obtain closed solutions of the above generating equations that determine the quantum action in the case of these gauge models, we assume a regularization that reduces all terms containing $\delta(0)$ to zero. Using this assumption we obtain a closed form of the vacuum functional and the corresponding transformations of extended BRST symmetry.

2. Freedman–Townsend Model

The theory of a non-abelian antisymmetric field $B^p_{\mu\nu}$, suggested by Freedman and Townsend [15], is described (in the first order formalism) by the action

$$S_0(A^p_\mu, B^p_{\mu\nu}) = \int d^4x\left(-\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}F^p_{\mu\nu}B^p_{\rho\sigma} + \frac{1}{2}A^p_\mu A^{p\mu}\right), \tag{2.1}$$

where A^p_μ is a vector field with the strength $F^p_{\mu\nu} = \partial_\mu A^p_\nu - \partial_\nu A^p_\mu + f^{pqr}A^q_\mu A^r_\nu$ (the coupling constant is absorbed into the structure coefficients f^{pqr}), the Levi-Civita tensor $\varepsilon^{\mu\nu\rho\sigma}$

is normalized as $\varepsilon^{0123} = 1$. Eliminating the auxiliary gauge field A^p_μ through its field equations leads to the more complicated action of the second order formalism.

The action (2.1) is invariant under the gauge transformations

$$\delta A^p_\mu = 0, \quad \delta B^p_{\mu\nu} = \mathcal{D}^{pq}_\mu \xi^q_\nu - \mathcal{D}^{pq}_\nu \xi^q_\mu \equiv \mathcal{R}^{pq}_{\mu\nu\alpha} \xi^{q\alpha}, \tag{2.2}$$

where ξ^p_μ are arbitrary parameters, and $\mathcal{D}^{pq}_\mu$ is the covariant derivative with potential A^p_μ ($\mathcal{D}^{pq}_\mu = \delta^{pq}\partial_\mu + f^{prq} A^r_\mu$).

The gauge transformations (2.2) form an abelian algebra with the generators $R^{pq}_{\mu\nu\alpha}$ possessing at the extremals of the action (2.1) the zero-eigenvectors $\mathcal{Z}^{pq}_\mu \equiv \mathcal{D}^{pq}_\mu$

$$\mathcal{R}^{pq}_{1\mu\nu} \equiv \mathcal{R}^{pr}_{\mu\nu\alpha} \mathcal{Z}^{rq\alpha} = \varepsilon_{\mu\nu\alpha\beta} f^{prq} \frac{\delta S_0}{\delta B^r_{\alpha\beta}}\,, \tag{2.3}$$

which, in their turn, are linearly independent. According to the generally accepted terminology, the model (2.1), (2.2) and (2.3) is an abelian gauge theory of first stage reducibility.

Note that the gauge structure of the Freedman–Townsend model [15] is similar to that of the Witten string [18]. The model [15] has been proved to be a convenient conceptual laboratory for the study [22] of the S-matrix unitarity in the framework of covariant quantization. Thus, in [22] it was shown that the application of the BV quantization rules to the model [15] leads to a physically unitary theory equivalent to a non-linear σ-model in $d = 4$. Note also that various aspects of the quantization of the Freedman–Townsend model in the framework of standard BRST symmetry have been discussed in [19, 23].

Now, let us consider the reducible gauge model (2.1), (2.2) and (2.3) in the framework of the modified triplectic formalism.

To this end, we first introduce the complete configuration space ϕ^A, which is constructed according to the standard prescriptions of the $Sp(2)$ covariant formalism [1] for reducible gauge theories. Namely, the space of the variables ϕ^A consists of the initial classical fields $A^{p\mu}$ and $B^{p\mu\nu}$, supplemented, firstly, by $Sp(2)$ doublets of Faddeev–Popov ghosts, C^{pa}_μ, introduced according to the number of the gauge parameters ξ^p_μ in eq. (2.2), secondly, by sets of first-stage ghost fields, C^{pab}, being symmetric $Sp(2)$ tensors, introduced according to the number of the gauge parameters ξ^p for the generators $R^{pq}_{1\mu\nu}$ in eq. (2.3), and, finally, by sets of auxiliary fields (Lagrange multipliers) B^p_μ, corresponding to the gauge parameters ξ^p_μ, and first-stage $Sp(2)$ doublets B^{pa}, corresponding to the parameters ξ^p. (Despite using the same letter B for the Lagrange multiplier fields as the physical antisymmetric tensor field $B_{\mu\nu}$ there should not appear any confusion since the former ones are Lorentz scalar or vector fields.)

The fileds ϕ^A of the complete configuration space take values in the adjoint representation of a non-abelian group; however, in what follows, the index $p = 1, \ldots, N$ will be omitted, e.g.

$$\phi^A = (A^\mu,\, B^{\mu\nu};\; B^\mu,\, B^a;\; C^{\mu a},\, C^{ab}).$$

The Grassmann parity of the fields ϕ^A is given by

$$\varepsilon(A^\mu) = \varepsilon(B^{\mu\nu}) = \varepsilon(B^\mu) = \varepsilon(C^{ab}) = 0, \qquad \varepsilon(B^a) = \varepsilon(C^{\mu a}) = 1.$$

In accordance with the quantization rules [6], the set of the fields ϕ^A is supplemented by the corresponding sets of the variables ϕ^*_{Aa}, π^{Aa} and $\bar{\phi}_A$

$$\begin{aligned}
\phi^*_{Aa} &= (A^*_{\mu a}, B^*_{\mu\nu a};\; B^*_{\mu a}, B^*_{a|b};\; C^*_{\mu a|b}, C^*_{a|bc}),\\
\pi^{Aa} &= (\pi^{\mu a}_{(A)}, \pi^{\mu\nu a}_{(B)};\; \pi^{\mu a}_{(B)}, \pi^{a|b}_{(B)};\; \pi^{\mu a|b}_{(C)}, \pi^{a|bc}_{(C)}),\\
\bar{\phi}_A &= (\bar{A}_\mu, \bar{B}_{\mu\nu};\; \bar{B}_\mu, \bar{B}_a;\; \bar{C}_{\mu a}, \bar{C}_{ab}),
\end{aligned}$$

as well as by the auxiliary variables λ^A

$$\lambda^A = (\lambda^\mu_{(A)},\, \lambda^{\mu\nu}_{(B)};\; \lambda^\mu_{(B)},\, \lambda^a_{(B)};\; \lambda^{\mu a}_{(C)},\, \lambda^{ab}_{(C)}),$$

with the following Grassmann parity:

$$\varepsilon(\phi^*_{Aa}) = \varepsilon(\pi^{Aa}) = \varepsilon(\phi^A) + 1, \quad \varepsilon(\bar{\phi}_A) = \varepsilon(\lambda^A) = \varepsilon(\phi^A). \tag{2.4}$$

A solution of the generating equations (1.1) for the quantum action S of the model in question can be found in a closed form:

$$\begin{aligned} S &= \int d^4x \left(-\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} B_{\rho\sigma} + \frac{1}{2} A_\mu A^\mu \right) \\ &+ \int d^4x \Big\{ B^*_{\mu\nu a}(\mathcal{D}^\mu C^{\nu a} - \mathcal{D}^\nu C^{\mu a}) - \varepsilon^{ab} C^*_{\mu a|b} B^\mu + \bar{B}_{\mu\nu}(\mathcal{D}^\mu B^\nu - \mathcal{D}^\nu B^\mu) \\ &+ C^*_{\mu a|b}\mathcal{D}^\mu C^{ab} - \varepsilon^{ab} C^*_{a|bc} B^c - \frac{1}{2} B^*_{\mu a}\mathcal{D}^\mu B^a + \bar{C}_{\mu a}\mathcal{D}^\mu B^a \\ &+ \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}(B^*_{\mu\nu a} \wedge B^*_{\rho\sigma b}) C^{ab} - \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}(B^*_{\mu\nu a} \wedge \bar{B}_{\rho\sigma}) B^a \Big\} \end{aligned} \tag{2.5}$$

where, after having omitted the gauge indices we use the notation $A^p B^p = AB$, $\mathcal{D}_\mu B = \partial_\mu B + A_\mu \wedge B$, $(A \wedge B)^p = f^{pqr} A^q B^r$.

A solution of the generating equation (1.2) for the gauge-fixing functional X can be represented as

$$\begin{aligned} X &= \int d^4x \Big\{ (\bar{B}_{\mu\nu} + \frac{\alpha}{2} B_{\mu\nu})\lambda^{\mu\nu}_{(B)} + \left(\bar{C}_{\mu a} - \beta\varepsilon_{ab} C^b_\mu\right)\lambda^{\mu a}_{(C)} \\ &+ \frac{\alpha}{4}\varepsilon_{ab}\pi^a_{(B)\mu\nu}\pi^{\mu\nu b}_{(B)} - \frac{\beta}{2}\varepsilon_{ab}\varepsilon_{cd}\pi_{(C)}{}^{a|c}_{\mu}\pi^{\mu b|d}_{(C)} \Big\}, \end{aligned} \tag{2.6}$$

where α and β are constant parameters.

Now, substituting the solutions of S, eq. (2.5), and X, eq. (2.6), into eq. (1.3), we obtain the corresponding vacuum functional, with the integrand being invariant under the following symmetry transformations (μ_a is a doublet of anticommuting constant parameters):

$$\begin{aligned} \delta A^\alpha &= \pi^{\alpha a}_{(A)}\mu_a, \\ \delta B^{\alpha\beta} &= \pi^{\alpha\beta a}_{(B)}\mu_a - \Big(\mathcal{D}^\alpha C^{\beta a} - \mathcal{D}^\beta C^{\alpha a} + \varepsilon^{\alpha\beta\gamma\delta} B^*_{\gamma\delta b} \wedge C^{ab} - \frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}\bar{B}_{\gamma\delta} \wedge B^a\Big)\mu_a, \\ \delta B^\alpha &= \pi^{\alpha a}_{(B)}\mu_a + \frac{1}{2}\mathcal{D}^\alpha B^a \mu_a, \\ \delta B^a &= \pi^{b|a}_{(B)}\mu_b, \\ \delta C^{\alpha a} &= \pi^{\alpha b|a}_{(C)}\mu_b - \Big(\varepsilon^{ab} B^\alpha + \mathcal{D}^\alpha C^{ab}\Big)\mu_b, \\ \delta C^{ab} &= \pi^{c|ab}_{(C)}\mu_c + \frac{1}{2}\varepsilon^{c\{a} B^{b\}}\mu_c, \\ \delta A^*_{\alpha a} &= \mu_a\Big(-\frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}\mathcal{D}^\beta B^{\gamma\delta} + A_\alpha - 2B^*_{\alpha\beta} \wedge C^{\beta a} - 2\bar{B}_{\alpha\beta} \wedge B^\beta \\ &\quad - C^*_{\alpha a|b} \wedge C^{ab} + \frac{1}{2} B^*_{\alpha a} \wedge B^a - \bar{C}_{\alpha a} \wedge B^a \Big), \\ \delta B^*_{\alpha\beta a} &= -\mu_a\Big(\frac{1}{4}\varepsilon_{\alpha\beta\gamma\delta} F^{\gamma\delta} + \frac{\alpha}{2}\lambda_{(B)\alpha\beta}\Big), \end{aligned}$$

$$
\begin{aligned}
\delta B^*_{\alpha a} &= \mu_a\Big(2\mathcal{D}^\beta \bar{B}_{\alpha\beta} - \varepsilon^{ab}C^*_{\alpha a|b}\Big),\\
\delta B^*_{a|b} &= \mu_a\Big(\varepsilon^{cd}C^*_{c|bd} + \frac{1}{2}\mathcal{D}^\alpha B^*_{\alpha b} - \mathcal{D}^\alpha \bar{C}_{\alpha b} - \frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}(B^*_{\alpha\beta b}\wedge \bar{B}_{\gamma\delta})\Big),\\
\delta C^*_{\alpha a|b} &= \mu_a\Big(2\mathcal{D}^\beta B^*_{\alpha\beta b} + \beta\varepsilon_{bd}\lambda^d_{(C)\alpha}\Big),\\
\delta C^*_{a|bc} &= \mu_a\Big(-\frac{1}{2}\mathcal{D}^\alpha \bar{C}_{\alpha\{b|c\}} + \frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}(B^*_{\alpha\beta\{b}\wedge B^*_{\gamma\delta|c\}})\Big),\\
\delta\pi^{\alpha a}_{(A)} &= -\varepsilon^{ab}\lambda^\alpha_{(A)}\mu_b,\\
\delta\pi^{\alpha\beta a}_{(B)} &= -\varepsilon^{ab}\lambda^{\alpha\beta}_{(B)}\mu_b + \varepsilon^{ab}\Big(\mathcal{D}^\alpha B^\beta - \mathcal{D}^\beta B^\alpha + \frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}(B^*_{\delta\gamma c}\wedge B^c)\Big)\mu_b,\\
\delta\pi^{\alpha a}_{(B)} &= -\varepsilon^{ab}\lambda^\alpha_{(B)}\mu_b,\\
\delta\pi^{a|b}_{(B)} &= -\varepsilon^{ac}\lambda^b_{(B)}\mu_c,\\
\delta\pi^{\alpha a|b}_{(C)} &= -\varepsilon^{ac}\lambda^{\alpha b}_{(C)}\mu_c + \varepsilon^{ac}\mathcal{D}^\alpha C^b\mu_c,\\
\delta\pi^{a|bc}_{(C)} &= -\varepsilon^{ad}\lambda^{bc}_{(C)}\mu_d,\\
\delta\bar{A}_\alpha &= \mu_a\varepsilon^{ab}A^*_{\alpha b},\\
\delta\bar{B}_{\alpha\beta} &= \mu_a\Big(\varepsilon^{ab}B^*_{\alpha\beta b} - \frac{\alpha}{2}\pi^a_{(B)\alpha\beta}\Big),\\
\delta\bar{B}_\alpha &= \mu_a\varepsilon^{ab}B^*_{\alpha b},\\
\delta\bar{B}_a &= \mu_c\varepsilon^{cb}B^*_{b|a},\\
\delta\bar{C}_{\alpha a} &= \mu_c\Big(\varepsilon^{cb}C^*_{\alpha b|a} + \beta\varepsilon_{ab}\pi^{c|b}_{(C)\alpha}\Big),\\
\delta\bar{C}_{ab} &= \mu_c\varepsilon^{cd}C^*_{d|ab}.
\end{aligned}
\tag{2.7}
$$

where symmetrization over $Sp(2)$ indices is taken as $A^{\{ab\}} = A^{ab} + A^{ba}$. Eqs. (2.7) realize the transformations of extended BRST symmetry of the vacuum functional in terms of the anticanonically conjugated pairs of variables $\{\phi^A,\, \phi^*_{Aa}\}$ and $\{\pi^{Aa},\, \bar{\phi}_A\}$.

Integrating in eq. (1.3) over the variables ϕ^*_{Aa}, π^{Aa}, $\bar{\phi}_A$ and λ^A, we represent the vacuum functional Z as an integral over the fields ϕ^A of the complete configuration space,

$$
Z = \int d\phi\,\Delta\exp\Big\{\frac{i}{\hbar}S^{(0)}_{\rm eff}(\phi)\Big\},
\tag{2.8}
$$

where

$$
\begin{aligned}
S^{(0)}_{\rm eff} &= \int d^4x\left(-\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}B_{\rho\sigma} + \frac{1}{2}A_\mu A^\mu\right)\\
&+ \int d^4x\left\{\frac{\alpha}{4}G^a_{\mu\nu}\mathcal{M}_{ab}\mathcal{K}^{b[\mu\nu][\rho\sigma]}_c G^c_{\rho\sigma} - \frac{\beta}{2}\varepsilon_{ab}\varepsilon_{cd}\Big(\mathcal{D}_\mu C^{ac}\Big)\Big(\mathcal{D}^\mu C^{bd}\Big)\right\}\\
&+ \int d^4x\left(\alpha B_\mu\mathcal{D}_\nu B^{\nu\mu} + \beta\varepsilon_{ab}B^a\mathcal{D}_\mu C^{\mu b} - \beta B_\mu B^\mu\right),
\end{aligned}
\tag{2.9}
$$

$$
\Delta = \int dB^*\exp\left\{\frac{2i}{\alpha\hbar}\int d^4x\, B^*_{0ia}\mathcal{M}^{ab}B^*_{0jb}\eta^{ij}\right\}.
\tag{2.10}
$$

In eq. (2.9) we have used the following notations:

$$
\mathcal{K}^{a[\mu\nu][\rho\sigma]}_b \equiv \frac{1}{2}\{\delta^a_b(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}) + \alpha\mathcal{C}^a_b\varepsilon^{\mu\nu\rho\sigma}\},
$$

$$G^a_{\mu\nu} \equiv \left(\mathcal{D}_\mu C^a_\nu - \mathcal{D}_\nu C^a_\mu\right) - \frac{\alpha}{4}\varepsilon_{\mu\nu\rho\sigma}\mathcal{B}^a B^{\rho\sigma}.$$

The matrix $\mathcal{M}_{ab}$ in (2.10) is the inverse of $\mathcal{M}^{ab}$

$$\mathcal{M}^{ab} \equiv \varepsilon^{ab} - \alpha^2 \mathcal{C}^a_c \mathcal{C}^b_d \varepsilon^{cd}, \quad \mathcal{M}^{ac}\mathcal{M}_{cb} = \delta^a_b,$$

where the matrices $\mathcal{C}^a_b$ and $\mathcal{B}^a$ are defined by

$$\mathcal{C}^a_b E \equiv \varepsilon_{bc} C^{ac} \wedge E, \quad \mathcal{B}^a E \equiv B^a \wedge E.$$

The functional $S^{(0)}_{\rm eff}$ in eq. (2.9) is the tree approximation to the gauge-fixed quantum action of the theory, while the functional Δ in eq. (2.10) can be considered as a contribution to the integration measure.

The integrand in eq. (2.8) is invariant under the following symmetry transformations:

$$\begin{aligned}
\delta A_\mu &= 0, \quad \delta B^a = 0, \\
\delta B^{\alpha\beta} &= -\varepsilon^{ab}\mathcal{M}_{bc}\mathcal{K}^{c[\alpha\beta][\gamma\delta]}_d G^d_{\gamma\delta}\mu_a, \\
\delta B^\alpha &= \frac{1}{2}\mathcal{D}^\alpha B^a \mu_a, \\
\delta C^{\alpha a} &= (\mathcal{D}^\alpha C^{ab} - \varepsilon^{ab} B^\alpha)\mu_b, \\
\delta C^{ab} &= \frac{1}{2} B^{\{a}\varepsilon^{b\}c}\mu_c.
\end{aligned} \tag{2.11}$$

The transformations (2.11) leave invariant both the action $S^{(0)}_{\rm eff}$ and the integration measure $d\phi\Delta$ in eq. (2.8),

$$\delta(d\phi) = d\phi\, \delta^4(0) \int d^4x \,{\rm Tr}\, \mathcal{W},$$

$$\delta\Delta = -\Delta\delta^4(0) \int d^4x \,{\rm Tr}\, \mathcal{W},$$

$$\delta\left(\exp\left\{\frac{i}{\hbar} S^{(0)}_{\rm eff}\right\}\right) = 0,$$

where the following notations have been used:

$$\mathcal{W} = -3\alpha^2 \varepsilon^{ab}(\mathcal{M}_{bc}\mathcal{C}^c_d \mathcal{B}^d)\mu_a\,, \quad {\rm Tr}\, \mathcal{W} \equiv \sum_{p=1}^{N} W^{pp}.$$

Consequently, eqs. (2.11) realize the transformations of extended BRST symmetry for the vacuum functional (2.8) in terms of the variables ϕ^A of the complete configuration space.

Note that, taking into account the action (2.9) and the contribution to the integration measure (2.10), the vacuum functional (2.8) obtained for the Freedman–Townsend model leads to the unitarity [20] of the physical S matrix (for discussions of the unitarity problem in the case of this model, see also [22, 19, 21]). For the first time covariant quantization of the Freedman-Townsend model in the framework of extended BRST invariance has been performed in [24].

3. W_2-gravity

The model of W_2-gravity [16] is described by the action

$$S_0(\varphi, h) = \frac{1}{2\pi}\int d^2z\left(\partial\varphi\bar{\partial}\varphi - h(\partial\varphi)^2\right), \tag{3.1}$$

where φ and h are bosonic classical fields defined on a space with complex coordinates, $(z, \bar{z})$, so that $\partial = \partial/\partial z$, $\bar{\partial} = \partial/\partial\bar{z}$.

The action (3.1) is invariant under the gauge transformations

$$\begin{aligned} \delta\varphi &= (\partial\varphi)\xi, \\ \delta h &= \bar{\partial}\xi - h\partial\xi + (\partial h)\xi \end{aligned} \tag{3.2}$$

with the parameter ξ.

The transformations (3.2) form a closed algebra,

$$\begin{aligned} &[\delta_{\xi(1)},\ \delta_{\xi(2)}] = \delta_{\xi(1,2)}\ , \\ &\xi_{(1,2)} = (\partial\xi_{(1)})\xi_{(2)} - (\partial\xi_{(2)})\xi_{(1)}. \end{aligned} \tag{3.3}$$

Note that the quantum properties of W_2-gravity considered within the BV method [14] have been discussed in [25, 26]. Recently, its quantization has been performed within the triplectic formalism [27].

Consider the gauge model (3.1), (3.2) and (3.3) in the framework of the modified triplectic quantization.

Let us introduce the complete configuration space ϕ^A, whose structure in the case of the model in question is determined by the rules of the $Sp(2)$ formalism for irreducible gauge theories. Thus, the space of the variables ϕ^A is constructed by supplementing the initial space of the fields (φ, h) with the doublet C^a, $\varepsilon(C^a) = 1$, of Faddeev–Popov ghosts, and the Lagrange multiplier B, $\varepsilon(B) = 0$, corresponding to the gauge parameter ξ in eq. (3.2).

The fields ϕ^A of the complete configuration space

$$\phi^A = (\varphi,\ h;\ B,\ C^a)$$

are supplemented by the sets of the variables ϕ^*_{Aa}, π^{Aa} and $\bar{\phi}_A$

$$\begin{aligned} \phi^*_{Aa} &= (\varphi^*_a,\ h^*_a;\ B^*_a,\ C^*_{a|b}), \\ \pi^{Aa} &= (\pi^a_{(\varphi)},\ \pi^a_{(h)};\ \pi^a_{(B)},\ \pi^{a|b}_{(C)}), \\ \bar{\phi}_A &= (\bar{\varphi},\ \bar{h};\ \bar{B},\ \bar{C}_a), \end{aligned}$$

as well as by the additional variables λ^A

$$\lambda^A = (\lambda_{(\varphi)},\ \lambda_{(h)};\ \lambda_{(B)},\ \lambda^a_{(C)}),$$

with the Grassmann parity given by eq. (2.4).

Consider the usual assumption of a regularization that reduces all terms containing $\delta(0)$ to zero. Then a functional that satisfies the generating equations (1.1) for the quantum action S in the case of the gauge model (3.1), (3.2) and (3.3) can be found in a closed form as follows

$$
\begin{aligned}
S &= \frac{1}{2\pi}\int d^2x\left(\partial\varphi\,\bar{\partial}\varphi - h(\partial\varphi)^2\right) \\
&+ \int d^2x\Big\{\varphi^*_a C^a\partial\varphi + h^*_a\left(\bar{\partial}C^a - h\partial C^a + C^a\partial h\right) \\
&+ \Big(\frac{1}{2}B^*_a - \bar{C}_a\Big)\Big[C^a\partial B - B\partial C^a \\
&+ \frac{1}{6}\varepsilon_{bd}\Big(C^{\{a}(\partial^2 C^{d\}})C^b - C^{\{a}(\partial C^{d\}})\partial C^b\Big)\Big] \\
&- C^*_{a|b}\Big(\varepsilon^{ab}B + \frac{1}{2}C^{\{a}\partial C^{b\}}\Big) \qquad (3.4)\\
&+ \bar{\varphi}\Big(B\partial\varphi + \frac{1}{2}\varepsilon_{ab}C^a\partial(C^b\partial\varphi)\Big) \\
&+ \bar{h}\Big(\bar{\partial}B - h\partial B + B\partial h\Big) \\
&+ \frac{1}{2}\varepsilon_{ab}\Big(C^a\partial(\bar{\partial}C^b - h\partial C^b + C^b\partial h) \\
&+ (\bar{\partial}C^b - h\partial C^b + C^b\partial h)\partial C^a\Big)\Big\}.
\end{aligned}
$$

A solution of the generating equations [6] determining the gauge-fixing functional X can be represented as

$$
\begin{aligned}
X &= \int d^2x\Big\{(\bar{\varphi} - \alpha\varphi - \beta h)\lambda_{(\varphi)} + (\bar{h} - \beta\varphi - \gamma h)\lambda_{(h)} \\
&- \frac{\alpha}{2}\varepsilon_{ab}\pi^a_{(\varphi)}\pi^b_{(\varphi)} - \beta\varepsilon_{ab}\pi^a_{(\varphi)}\pi^b_{(h)} - \frac{\gamma}{2}\varepsilon_{ab}\pi^a_{(h)}\pi^b_{(h)}\Big\}, \qquad (3.5)
\end{aligned}
$$

where α, β and γ are constant parameters.

The vacuum functional (1.3) corresponding to the solutions (3.4) and (3.5) of the generating equations [6] that determine the quantum action S and the gauge-fixing functional X is invariant under the following transformations of extended BRST symmetry, expressed (for simplicity) in terms of the derivatives of S:

$$
\begin{aligned}
\delta\phi^A &= \Big(\pi^{Aa} - \frac{\delta S}{\delta\phi^*_{Aa}}\Big)\mu_a, \\
\delta\varphi^*_a &= \mu_a\Big(\frac{\delta S}{\delta\varphi} + \alpha\lambda_{(\varphi)} + \beta\lambda_{(h)}\Big), \\
\delta h^*_a &= \mu_a\Big(\frac{\delta S}{\delta h} + \beta\lambda_{(\varphi)} + \gamma\lambda_{(h)}\Big), \\
\delta B^*_a &= \mu_a\frac{\delta S}{\delta B}, \\
\delta C^*_{a|b} &= \mu_a\frac{\delta S}{\delta C^b}, \\
\delta\pi^a_{(\varphi)} &= \varepsilon^{ab}\Big(\frac{\delta S}{\delta\bar{\varphi}} - \lambda_{(\varphi)}\Big)\mu_b, \\
\delta\pi^a_{(h)} &= \varepsilon^{ab}\Big(\frac{\delta S}{\delta\bar{h}} - \lambda_{(h)}\Big)\mu_b, \\
\delta\pi^a_{(B)} &= \varepsilon^{ab}\frac{\delta S}{\delta\bar{B}}\mu_b,
\end{aligned}
$$

$$
\begin{aligned}
\delta\pi^{a|b}_{(C)} &= \varepsilon^{ac}\frac{\delta S}{\delta\bar{C}_b}\mu_c,\\
\delta\bar{\varphi} &= \mu_a(\varepsilon^{ab}\varphi^*_b+\alpha\pi^a_{(\varphi)}+\beta\pi^a_{(h)}),\\
\delta\bar{h} &= \mu_a(\varepsilon^{ab}h^*_b+\beta\pi^a_{(\varphi)}+\gamma\pi^a_{(h)}),\\
\delta\bar{B} &= \mu_a\varepsilon^{ab}B^*_b,\\
\delta\bar{C}_a &= \mu_a\varepsilon^{bd}C^*_{d|b}.
\end{aligned}
\tag{3.6}
$$

Substituting the solutions (3.4) and (3.5) that determine the quantum action S and the gauge-fixing functional X into eq. (1.3), and integrating out the variables ϕ^*_{Aa}, π^{Aa}, $\bar{\phi}_A$ and λ^A, we express the vacuum functional Z as an integral over the fields ϕ^A of the complete configuration space,

$$
Z=\int d\phi\,\exp\left\{\frac{i}{\hbar}S_{\rm eff}(\phi)\right\},
\tag{3.7}
$$

where $S_{\rm eff}$ is the gauge-fixed quantum action

$$
\begin{aligned}
S_{\rm eff} &= \frac{1}{2\pi}\int d^2x\left(\partial\varphi\bar{\partial}\varphi-h(\partial\varphi)^2\right)\\
&+ \int d^2x\Big[(\alpha\varphi+\beta h)B\partial\varphi+(\beta\varphi+\gamma h)(\bar{\partial}B-h\partial B+B\partial h)\Big]\\
&+ \frac{1}{2}\varepsilon_{ab}\int d^2x\Big[\Big(\alpha C^b\partial\varphi+\beta(\bar{\partial}C^b-h\partial C^b+C^b\partial h)\Big)C^a\partial\varphi\\
&- (\alpha\varphi+\beta h)C^a\partial(C^b\partial\varphi)\\
&+ \Big(\beta C^b\partial\varphi+\gamma(\bar{\partial}C^b-h\partial C^b+C^b\partial h)\Big)(\partial C^a-h\partial C^a+C^a\partial h)\\
&- (\beta\varphi+\gamma h)\Big((\bar{\partial}C^b-h\partial C^b+C^b\partial h)\partial C^a\\
&+ C^a\partial(\bar{\partial}C^b-h\partial C^b+C^b\partial h)\Big)\Big].
\end{aligned}
\tag{3.8}
$$

The quantum action $S_{\rm eff}$ (3.8) and the integration measure $d\phi$ in eq. (3.7) are invariant under the following transformations:

$$
\begin{aligned}
\delta\varphi &= C^a\partial\varphi\;\mu_a,\\
\delta h &= \left(\bar{\partial}C^a-h\partial C^a+C^a\partial h\right)\mu_a,\\
\delta B &= \frac{1}{2}(C^a\partial B-B\partial C^a)\mu_a+\frac{1}{12}\varepsilon_{bd}\Big(C^{\{a}\partial^2C^{d\}}C^b-C^{\{a}\partial C^{d\}}\partial C^b\Big)\mu_a,\\
\delta C^a &= \Big(\varepsilon^{ab}B-\frac{1}{2}C^{\{a}\partial C^{b\}}\Big)\mu_b.
\end{aligned}
\tag{3.9}
$$

Thus we conclude that eqs. (3.9) realize the transformations of extended BRST symmetry for the vacuum functional (3.7) in terms of the variables of the complete configuration space.

4. Two-dimensional Gravity with Dynamical Torsion

The theory of two-dimensional gravity with dynamical torsion is described in terms of the zweibein and Lorentz connection (e^i_μ,ω_μ) by the action [17]

$$
S_0(e^i_\mu,\omega_\mu)=\int d^2x\;e\Big(\frac{1}{16\alpha}R_{\mu\nu}{}^{ij}R^{\mu\nu}{}_{ij}-\frac{1}{8\beta}T_{\mu\nu}{}^iT^{\mu\nu}{}_i-\gamma\Big),
\tag{4.1}
$$

where α, β and γ are constant parameters. In eq. (4.1), the Latin indices are lowered with the help of the Minkowski metric η_{ij} $(+,-)$, and the Greek indices, with the help of the metric tensor $g_{\mu\nu} = \eta_{ij} e^i_\mu e^j_\nu$. Besides, the following notations are used:

$$\begin{aligned} e &= \det e^i_\mu \,, \\ R_{\mu\nu}{}^{ij} &= \varepsilon^{ij} \partial_\mu \omega_\nu - (\mu \leftrightarrow \nu), \\ T_{\mu\nu}{}^i &= \partial_\mu e^i_\nu + \varepsilon^{ij} \omega_\mu e_{\nu j} - (\mu \leftrightarrow \nu), \end{aligned}$$

where ϵ^{ij} is a constant antisymmetric tensor, $\epsilon^{01} = -1$.

Note that the model (4.1) is the most general theory of two-dimensional R^2-gravity with independent dynamical torsion that leads to second-order equations of motion for the zweibein and Lorentz connection. Thus, supplementing the action eq. (4.1) by the Einstein–Hilbert term eR would not affect the classical field equations, since in two dimensions it reduces to a trivial total divergence.

Originally, the action (4.1) was proposed [28] in the context of bosonic string theory, where it was used to describe the dynamics of string geometry. There, moreover, it was proved that the string with dynamical geometry has no critical dimension.

An attractive feature of the model (4.1) is its complete classical integrability. The corresponding equations of motion have been studied in conformal [17, 29] and in light-cone [30] gauges. It was established that this model contains solutions with constant curvature and zero torsion, thus incorporating several other two-dimensional gravity models [31], whose actions, however, do not have a purely geometric interpretation.

The action (4.1) is invariant under the local Lorentz rotations of the zweibein e^i_μ, which infinitesimally implies the gauge transformations

$$\delta_\zeta e^i_\mu = \varepsilon^{ij} e_{\mu j} \zeta \qquad \delta_\zeta \omega_\mu = -\partial_\mu \zeta \tag{4.2}$$

with the parameter ζ. Similarly, the general coordinate invariance of eq. (4.1) leads to the gauge transformations

$$\delta_\xi e^i_\mu = e^i_\nu \partial_\mu \xi^\nu + (\partial_\nu e^i_\mu) \xi^\nu, \qquad \delta_\xi \omega_\mu = \omega_\nu \partial_\mu \xi^\nu + (\partial_\nu \omega_\mu) \xi^\nu \tag{4.3}$$

with the parameters ξ^μ. The gauge transformations (4.2) and (4.3) form a closed algebra

$$\begin{aligned} [\delta_{\zeta(1)},\ \delta_{\zeta(2)}] &= 0\,, \\ [\delta_{\xi(1)},\ \delta_{\xi(2)}] &= \delta_{\xi(1,2)}\,, \\ [\delta_\zeta,\ \delta_\xi] &= \delta_{\zeta'}\,, \end{aligned} \tag{4.4}$$

where

$$\xi^\mu{}_{(1,2)} = (\partial_\nu \xi^\mu{}_{(1)}) \xi^\nu{}_{(2)} - (\partial_\nu \xi^\mu{}_{(2)}) \xi^\nu{}_{(1)}\,, \quad \zeta^{'} = (\partial_\mu \zeta) \xi^\mu .$$

Note that in Ref. [32] a gauge model classically equivalent to (4.1), (4.2), (4.3) and (4.4) was proposed by means of introducing additional field variables; however, in this equivalent formulation the algebra of the corresponding gauge transformations closes only on-shell.

The Hamiltonian structure of the gauge symmetries of the original model was studied in Ref. [33], and its canonical quantization, in ef. [34]. Quantum properties of that theory were also discussed in Ref. [35].

Consider the gauge model (4.1), (4.2), (4.3) and (4.4) in the framework of the modified triplectic formalism [6].

The complete configuration space ϕ^A, constructed by the rules of the $Sp(2)$ covariant quantization of irreducible theories, consists of the initial classical fields (e^i_μ, ω_μ), the doublets of the Faddeev–Popov ghosts (C^a, $C^{\mu a}$) and the Lagrangian multipliers (B, B^μ) introduced according to the number of the gauge parameters in eqs. (4.2) and (4.3), i.e. ζ and ξ^μ, respectively. The Grassmann parity of the fields ϕ^A

$$\phi^A = (e^i_\mu,\ \omega_\mu;\ B,\ B^\mu;\ C^a,\ C^{\mu a})$$

is given by

$$\varepsilon(e^i_\mu) = \varepsilon(\omega_\mu) = \varepsilon(B) = \varepsilon(B^\mu) = 0, \qquad \varepsilon(C^a) = \varepsilon(C^{\mu a}) = 1.$$

The fields ϕ^A of the complete configuration space are supplemented by the sets of the variables ϕ^*_{Aa}, π^{Aa}, $\bar{\phi}_A$ and λ^A

$$\begin{aligned}
\phi^*_{Aa} &= (e^{*\mu}_{ia},\ \omega^{*\mu}_a;\ B^*_a,\ B^*_{\mu a};\ C^*_{a|b},\ C^*_{\mu a|b}),\\
\pi^{Aa} &= (\pi^{ia}_{(e)\mu},\ \pi^a_{(\omega)\mu};\ \pi^a_{(B)},\ \pi^{\mu a}_{(B)};\ \pi^{a|b}_{(C)},\ \pi^{\mu a|b}_{(C)}),\\
\bar{\phi}_A &= (\bar{e}^\mu_i,\ \bar{\omega}^\mu;\ \bar{B},\ \bar{B}_\mu;\ \bar{C}_a,\ \bar{C}_{\mu a}),\\
\lambda^A &= (\lambda^i_{(e)\mu},\ \lambda_{(\omega)\mu};\ \lambda_{(B)},\ \lambda^\mu_{(B)};\ \lambda^a_{(C)},\ \lambda^{\mu a}_{(C)}).
\end{aligned}$$

In the framework of the previously used assumption that $\delta(0)$, a functional that satisfies the generating equations (1.1) for the quantum action S in the case of the gauge model (4.1), (4.2), (4.3) and (4.4) can be found in a closed form,

$$\begin{aligned}
S &= \int d^2x\ e\Big(\frac{1}{16\alpha}R_{\mu\nu}{}^{ij}R^{\mu\nu}{}_{ij} - \frac{1}{8\beta}T_{\mu\nu}{}^iT^{\mu\nu}{}_i - \gamma\Big)\\
&+ \int d^2x\Big\{e^{*\mu}_{ia}\left(\varepsilon^{ij}e_{\mu j}C^a + C^{\lambda a}\partial_\lambda e^i_\mu + e^i_\lambda\partial_\mu C^{\lambda a}\right)\\
&+ \omega^{*\mu}_a\left(-\partial_\mu C^a + C^{\lambda a}\partial_\lambda\omega_\mu + \omega_\lambda\partial_\mu C^{\lambda a}\right)\\
&+ \Big(\frac{1}{2}B^*_a - \bar{C}_a\Big)\Big[(C^{\mu a}\partial_\mu B - B^\mu\partial_\mu C^a)\\
&+ \frac{1}{6}\varepsilon_{bd}(C^{\lambda\{a}\partial_\lambda C^{\mu d\}}\partial_\mu C^b - C^{\mu b}\partial_\mu C^{\lambda\{a}\partial_\lambda C^{d\}})\Big]\\
&+ \Big(\frac{1}{2}B^*_{\mu a} - \bar{C}_{\mu a}\Big)\Big[(C^{\lambda a}\partial_\lambda B^\mu - B^\lambda\partial_\lambda C^{\mu a})\\
&+ \frac{1}{6}\varepsilon_{bd}(C^{\sigma\{a}\partial_\sigma C^{\lambda d\}}\partial_\lambda C^{\mu b} - C^{\lambda b}\partial_\lambda C^{\sigma\{a}\partial_\sigma C^{\mu d\}})\Big]\\
&- C^*_{a|b}\left(\varepsilon^{ab}B + \frac{1}{2}C^{\mu\{a}\partial_\mu C^{b\}}\right) - C^*_{\mu a|b}\left(\varepsilon^{ab}B^\mu + \frac{1}{2}C^{\lambda\{a}\partial_\lambda C^{\mu b\}}\right)\\
&+ \bar{e}^\mu_i\Big[\varepsilon^{ij}Be_{\mu j} + B^\lambda\partial_\lambda e^i_\mu + e^i_\lambda\partial_\mu B^\lambda\\
&+ \frac{1}{2}\varepsilon_{ab}\Big((e^i_\mu C^b + \varepsilon^{ij}C^{\lambda b}\partial_\lambda e_{\mu j} + \varepsilon^{ij}e_{\lambda j}\partial_\mu C^{\lambda b})C^a\\
&- C^{\lambda a}\partial_\lambda(\varepsilon^{ij}e_{\mu j}C^b + C^{\sigma b}\partial_\sigma e^i_\mu + e^i_\sigma\partial_\mu C^{\sigma b})\\
&+ (\varepsilon^{ij}e_{\lambda j}C^b + (\partial_\sigma e^i_\lambda)C^{\sigma b} + e^i_\sigma\partial_\lambda C^{\sigma b})\partial_\mu C^{\lambda a}\Big)\Big]\\
&+ \bar{\omega}^\mu\Big[-\partial_\mu B + B^\lambda\partial_\lambda\omega_\mu + \omega_\lambda\partial_\mu B^\lambda\\
&- \frac{1}{2}\varepsilon_{ab}\Big(C^{\lambda a}\partial_\lambda(C^{\sigma b}\partial_\sigma\omega_\mu + \omega_\sigma\partial_\mu C^{\sigma b} - \partial_\mu C^b)\\
&- (C^{\sigma b}\partial_\sigma\omega_\lambda + \omega_\sigma\partial_\lambda C^{\sigma b} - \partial_\lambda C^b)\partial_\mu C^{\lambda a}\Big)\Big].
\end{aligned} \tag{4.5}$$

A solution of the generating equations determining the gauge-fixing functional X can be chosen as

$$
\begin{aligned}
X &= \int d^2x \Big\{ \Big(\bar{e}^{\mu}_{i} - p\eta^{\mu\nu}\eta_{ij}e^{j}_{\nu}\Big)\lambda^{i}_{(e)\mu} + \Big(\bar{\omega}^{\mu} - q\eta^{\mu\nu}\omega_{\nu}\Big)\lambda_{(\omega)\mu} \\
&- \frac{p}{2}\varepsilon_{ab}\eta_{ij}\eta^{\mu\nu}\pi^{ia}_{(e)\mu}\pi^{jb}_{(e)\nu} - \frac{q}{2}\varepsilon_{ab}\eta^{\mu\nu}\pi^{a}_{(\omega)\mu}\pi^{b}_{(\omega)\nu}\Big\},
\end{aligned} \tag{4.6}
$$

where p, q are constant parameters, and $\eta^{\mu\nu}$ $(+,-)$ is the metric of the two-dimensional Minkowski space.

Substituting the explicit solutions for the quantum action S (4.5) and the gauge-fixing functional X (4.6) into the expression that determines the vacuum functional Z and performing integration over the variables ϕ^{*}_{Aa}, π^{Aa}, $\bar{\phi}_A$ and λ^A, we obtain Z in the form (3.7) with the gauge-fixed quantum action $S_{\rm eff}$

$$
\begin{aligned}
S_{\rm eff} &= \int d^2x\, e\Big(\frac{1}{16\alpha}R_{\mu\nu}{}^{ij}R^{\mu\nu}{}_{ij} - \frac{1}{8\beta}T_{\mu\nu}{}^{i}T^{\mu\nu}{}_{i} - \gamma\Big) \\
&+ \int d^2x \Big\{ q\eta^{\mu\nu}(\partial_\mu\omega_\nu)B + \eta^{\lambda\nu}\Big[p\Big(e_{\lambda i}\partial_\mu e^{i}_{\nu} - \partial_\lambda(e_{\mu i}e^{i}_{\nu})\Big) \\
&+ q\Big(\omega_\lambda\partial_\mu\omega_\nu - \partial_\lambda(\omega_\mu\omega_\nu)\Big)\Big]B^{\mu}\Big\} \\
&+ \frac{1}{2}\varepsilon_{ab}\int d^2x \Big\{ p\Big(\eta^{\mu\nu}(e^{i}_{\lambda}\partial_\nu C^{\lambda a} - e^{i}_{\nu}\partial_\lambda C^{\lambda a}) \\
&+ \eta^{\lambda\nu}e^{i}_{\nu}\partial_\lambda C^{\mu a}\Big)\Big(\varepsilon_{ij}C^{b}e^{j}_{\mu} - \eta_{ij}(C^{\sigma b}\partial_\sigma e^{j}_{\mu} + e^{j}_{\sigma}\partial_\mu C^{\sigma b})\Big) \\
&- q\Big(\eta^{\mu\nu}(\partial_\nu C^{a} - \omega_\lambda\partial_\nu C^{\lambda a} + \omega_\nu\partial_\lambda C^{\lambda a}) \\
&- \eta^{\lambda\nu}\omega_\nu\partial_\lambda C^{\mu a}\Big)(\partial_\mu C^{b} - C^{\sigma b}\partial_\sigma\omega_\mu - \omega_\sigma\partial_\mu C^{\sigma b})\Big\}.
\end{aligned} \tag{4.7}
$$

The quantum action $S_{\rm eff}$ (4.7) and the integration measure $d\phi^A$ in the functional integral are invariant under the following symmetry transformations:

$$
\begin{aligned}
\delta e^{i}_{\sigma} &= \Big(\varepsilon^{ij}e_{\sigma j}C^{a} + C^{\lambda a}\partial_\lambda e^{i}_{\sigma} + e^{i}_{\lambda}\partial_\sigma C^{\lambda a}\Big)\,\mu_a, \\
\delta\omega_\sigma &= \Big(-\partial_\sigma C^{a} + C^{\lambda a}\partial_\lambda\omega_\sigma + \omega_\lambda\partial_\sigma C^{\lambda a}\Big)\,\mu_a, \\
\delta B &= \frac{1}{2}\Big(C^{\sigma a}\partial_\sigma B - B^{\sigma}\partial_\sigma C^{a} + \frac{1}{6}\varepsilon_{bd}(C^{\lambda\{a}\partial_\lambda C^{\sigma d\}}\partial_\sigma C^{b} \\
&\quad - C^{\sigma b}\partial_\sigma(C^{\lambda\{a}\partial_\lambda C^{d\}})\Big)\mu_a, \\
\delta B^{\sigma} &= \frac{1}{2}\Big(C^{\lambda a}\partial_\lambda B^{\sigma} - B^{\lambda}\partial_\lambda C^{\sigma a} + \frac{1}{6}\varepsilon_{bd}(C^{\rho\{a}\partial_\rho C^{\lambda d\}}\partial_\lambda C^{\sigma b} \\
&\quad - C^{\lambda b}\partial_\lambda(C^{\rho\{a}\partial_\rho C^{\sigma d\}})\Big)\mu_a, \\
\delta C^{a} &= \Big(\varepsilon^{ab}B - \frac{1}{2}C^{\sigma\{a}\partial_\sigma C^{b\}}\Big)\mu_b, \\
\delta C^{\sigma a} &= \Big(\varepsilon^{ab}B^{\sigma} - \frac{1}{2}C^{\lambda\{a}\partial_\lambda C^{\sigma b\}}\Big)\mu_b.
\end{aligned} \tag{4.8}
$$

Eqs. (4.8) realize the corresponding transformations of extended BRST symmetry for the vacuum functional in terms of the variables ϕ^A of the complete configuration space.

5. Conclusion

In this paper we have exemplified the method of modified triplectic quantization [6] on the basis of several gauge theory models. Thus, we have considered the model [15] of non-abelian antisymmetric tensor field (Freedman–Townsend model), the model [16] of W_2-gravity, and the model [17] of two-dimensional gravity with dynamical torsion. For the models in question we have found manifest solutions of the generating equations that determine the quantum action S and the gauge-fixing functional X in the framework of the modified triplectic formalism [6].

The above solutions are expressed in terms of the variables ϕ^A, ϕ^*_{Aa} and π^{Aa}, $\bar{\phi}_A$ anticanonically conjugated in the sense of the extended antibrackets [2, 6], as well as in terms of the additional variables λ^A that serve to parametrize the gauge-fixing functional X. Using the solutions for S and X, we have obtained the vacuum functional and explicitly constructed the corresponding transformations [6] of extended BRST symmetry in terms of the anticanonically conjugated variables. Finally, we have obtained manifest expressions for the quantum action S_{eff} that results from integrating out the variables ϕ^*_{Aa}, π^{Aa}, $\bar{\phi}_A$ and λ^A in the functional integral, and have constructed the corresponding transformations of extended BRST symmetry in terms of the variables ϕ^A of the complete configuration space.

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